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Classical spin dynamics of four interacting magnetic particles on a ring

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Abstract

In the present work, we study the deterministic spin dynamics of four interacting magnetic particles, with both dipolar and exchange interactions in the presence of an applied magnetic field, by means of the Landau–Lifshitz equation without the dissipation term. In particular, we analyze the ring geometrical configuration with periodic boundary conditions for the exchange coupling. In addition, we explore the parameter space by numerically calculating some bifurcation diagrams. Due to the strength ratio of interactions, two time scales appear. Finally, we find that the total magnetization is not conserved and it has a strong dependence on the control parameters.

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Keywords: Spin dynamics; Dipolar interaction; Non-uniform magnetization

1. Introduction

The increasing demand for improved recording media has impelled worldwide research for the development of magnetic nanoparticles applications. In this context, for the dynamics of interacting magnetic particles, considered as monodomains, it is essential to model spintronic devices [1]; hence, a detailed study of a simple interacting magnetic system is really important and in order.

Theoretical studies for two, three, and four magnetic particles interacting through exchange interaction were performed in Refs. [2–5]. The exact solution for classical spin dynamics of two magnetic particles was calculated in Ref. [2]; finding that the dynamical behavior is oscillatory. Also, the dynamical behavior of two magnetic particles in the presence of a strong magnetic for both classical and quantum description was evaluated in Ref. [3], determining that the total magnetization makes precessions around the applied magnetic field. The exact solution of the spin dynamics of three magnetic particles and its corresponding autocorrelation function were established for both linear and triangular geometrical configurations, considering a null magnetic field [4]. The ring geometrical configuration for four magnetic particles, assumed as classical Heisenberg spins, was analyzed in Ref. [5]; in this authoritative work the time evolution of each one of the spins and their corresponding time correlation functions were solved explicitly for any temperature.

On the other hand, the problem of interacting magnetic particles, coupled by the long-range magneto-dipole interaction, is of substantial interest and importance due to possible applications of nano-patterned magnetic media for magnetic memory systems. The main interaction between different magnetic nanoparticles, that are used to store the bits of information, is the magneto-dipole interaction, and the dynamics of such dipole-coupled systems is not well understood. In this framework, the dynamical behavior of two particles interacting through dipole–dipole interaction was analyzed in Ref. [6]. The authors conclude that, due to dipolar interaction the total magnetization is not a constant; furthermore, the total magnetization is a fluctuating time dependent function.

In the present work, we analyze in detail the deterministic behavior of four interacting magnetic particles, via dipole–dipole interaction and via exchange interaction, in the presence of an applied magnetic field. In particular, we analyze the ring geometrical configuration with periodic

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boundary conditions for the exchange coupling. We consider different regimens of the parameters for understanding corresponding magnetic behaviors. It has been remarked [7] that, when a magnetic system subjected to short field pulses and its dynamics is fast then the dissipative effects can be neglected; hence for short time interval we have here applied this feature. The paper is arranged in the following way: In Section 2 the theoretical model is described. In Section 3 the numerical results are discussed. Finally, conclusions are presented in Section 4.

2. Theoretical model

Let us consider a system of N magnetic particles and assume that each particle can be represented by a magnetic monodomain. The temporal evolution of this system can be modeled by the of Landau–Lifshitz (LL) equation [8], and in the absence of damping, it can be written as

$$\frac{\mathrm{d}\mathbf{m}^i}{\mathrm{d}t} = \gamma \mathbf{m}^i \times \mathbf{H}^i_{\mathrm{eff}}, \tag{1}$$

where $\mathbf{m}^i$ is an individual magnetic moment with $i = (1, \ldots, N)$ and γ is an effective gyromagnetic ratio. The corresponding ith effective field, $\mathbf{H}^i_{\mathrm{eff}}$, is given by $\mathbf{H}^i_{\mathrm{eff}} = -\nabla_{\mathbf{m}^i} H$ where H represents the appropriate Hamiltonian for the system.

At this point, we would like to remark that the structure of Eq. (1) has an intrinsic relationship with the Nambu's equation governing the dynamics for a triplet of canonical variables with two motion constants [9]; in the case of a single magnetic moment, the triplet of canonical variables is given by $\mathbf{m}$ and the two motion constants are the Hamiltonian, H, and the magnitude of the magnetic moment, $|\mathbf{m}|$. In addition, due to the individual magnetization magnitudes being constant, the system (1) can be reduced to a system with $2N$ coupled differential equations. In order to describe this characteristic we use the following transformation: $\mathbf{m}^i = (f(m^i_z)\cos\xi^i, f(m^i_z)\sin\xi^i, m^i_z)^{\mathrm{T}}$ with $f(m^i_z) = \sqrt{1-(m^i_z)^2}$, so if we considered the $2N$ dimensional vector $\boldsymbol{\eta} = (m^1_z, \ldots m^N_z; \xi^1, \ldots, \xi^N)^{\mathrm{T}} = (\mathbf{m}_z; \boldsymbol{\xi})^{\mathrm{T}}$ as the vector of canonical variables and conjugate momentum vector , the equation for $\boldsymbol{\eta}$ can be cast in the form

$$\frac{\mathrm{d}\boldsymbol{\eta}}{\mathrm{d}t} = \begin{pmatrix} \mathbf{0} & -\mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{pmatrix} \nabla_{\boldsymbol{\eta}} H(\boldsymbol{\eta}), \tag{2}$$

where $\mathbf{I}$ is the N dimensional identity matrix. Eq. (2) is the Hamilton's equation of motion for $\boldsymbol{\eta}$. This representation of Eq. (1) is an elegant form to present the classical spin dynamics; however it is not useful for numerical integration.

In this work we analyze four interacting magnetic particles on a ring, including dipolar and exchange interactions in the presence of an external field; therefore, the Hamiltonian has the following form:

$$H = -\sum_{i=1}^{4}\left(\mathbf{H}\cdot\mathbf{m}^i + J\mathbf{m}^i\cdot\mathbf{m}^{i+1}\right) + \sum_{i\neq k} r_{ik}^{-3}\left(\mathbf{m}^i\cdot\mathbf{m}^k - 3\left(\mathbf{m}^i\cdot\hat{\mathbf{r}}_{ik}\right)\left(\mathbf{m}^k\cdot\hat{\mathbf{r}}_{ik}\right)\right), \tag{3}$$

where $\mathbf{H}$ represents the external magnetic field, J is the exchange coupling constant and $\mathbf{r}_{ik} = \mathbf{r}_i - \mathbf{r}_k$, with $\mathbf{r}_i$ being the position vector of the i-magnetic moment. Hence, for periodic boundary conditions, inserting Eq. (3) into Eq. (1) produces the corresponding LL equations of the system:

$$\frac{\mathrm{d}\mathbf{m}^i}{\mathrm{d}t} = \gamma\mathbf{m}^i \times \left(\mathbf{H} + J\left(\mathbf{m}^{i-1} + \mathbf{m}^{i+1}\right) - \sum_k r_{ik}^{-3}\left(\mathbf{m}^k - 3\left(\mathbf{m}^k\cdot\hat{\mathbf{r}}_{ik}\right)\hat{\mathbf{r}}_{ik}\right)\right). \tag{4}$$

We note that the corresponding LL equations have a nonlinear coupling due to the interaction terms; and the system presents two time scales depending on the magnitude of the magnetic interactions, because they are of different nature. Furthermore, when the distance between

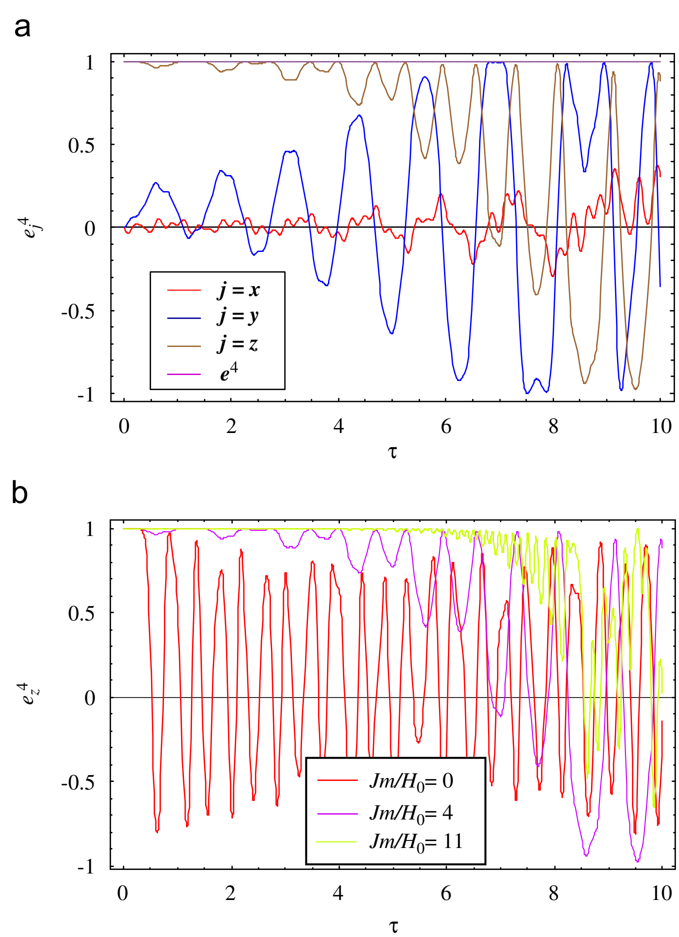

Fig. 1. (a) Cartesian components of the fourth magnetic particle, e^4_j, as a function of time for $h = 1$, $m/(d^3H_0) = 15.256$ and $J/(mH_0) = 4$. (b) z component of the fourth magnetic particle, e^i_z, as a function of time for different values of J at $h = 1$ and $m/(d^3H_0) = 15.256$.

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the magnetic moment increases, the dominant terms in Eq. (4) are due to the exchange and the external applied field, in that case the total magnetization precesses around the applied field direction and the individual magnetizations precesses around another direction which is defined by the sum of the external field and the exchange coupling constant times the total magnetization. Besides, Eq. (4) has four constraints: the individual magnetization magnitudes are constant. These constraints have an important consequence in the dynamical behavior: the global dynamics of each magnetic moment is reduced to a spherical section.

As a final comment of this section, we remark that due to the dipolar interaction, the magnitude of the total magnetization is not stationary. With the purpose of elucidating this property we can write $\mathbf{M} = \sum_{i=1}^{4} \mathbf{m}^i$, so the dynamics of $\mathbf{M}$ obeys the equation

$$\frac{\mathrm{d}\mathbf{M}}{\mathrm{d}t} = \gamma \mathbf{M} \times \mathbf{H} + \mathbf{F}. \quad (5)$$

Eq. (5) describes the behavior of an equivalent single magnetic moment in the presence of an applied magnetic field and an extra fluctuating term, $\mathbf{F} = \mathbf{F}\{\mathbf{m}^i\}$, which is a function of $\mathbf{m}^i$. Since, the set of nonlinearities $\mathbf{m}^i \times \hat{\mathbf{r}}_{ik}(\mathbf{m}^k \cdot \hat{\mathbf{r}}_{ik})$, in Eq. (4), generate $\mathbf{F}$, consequently Eq. (5) will have a different behavior with respect to the pure precessional dynamics; actually, this extra field can be interpreted as a time dependent fluctuating field. Also, $\mathbf{F}$ has a different structure from the Gilbert dissipation term:

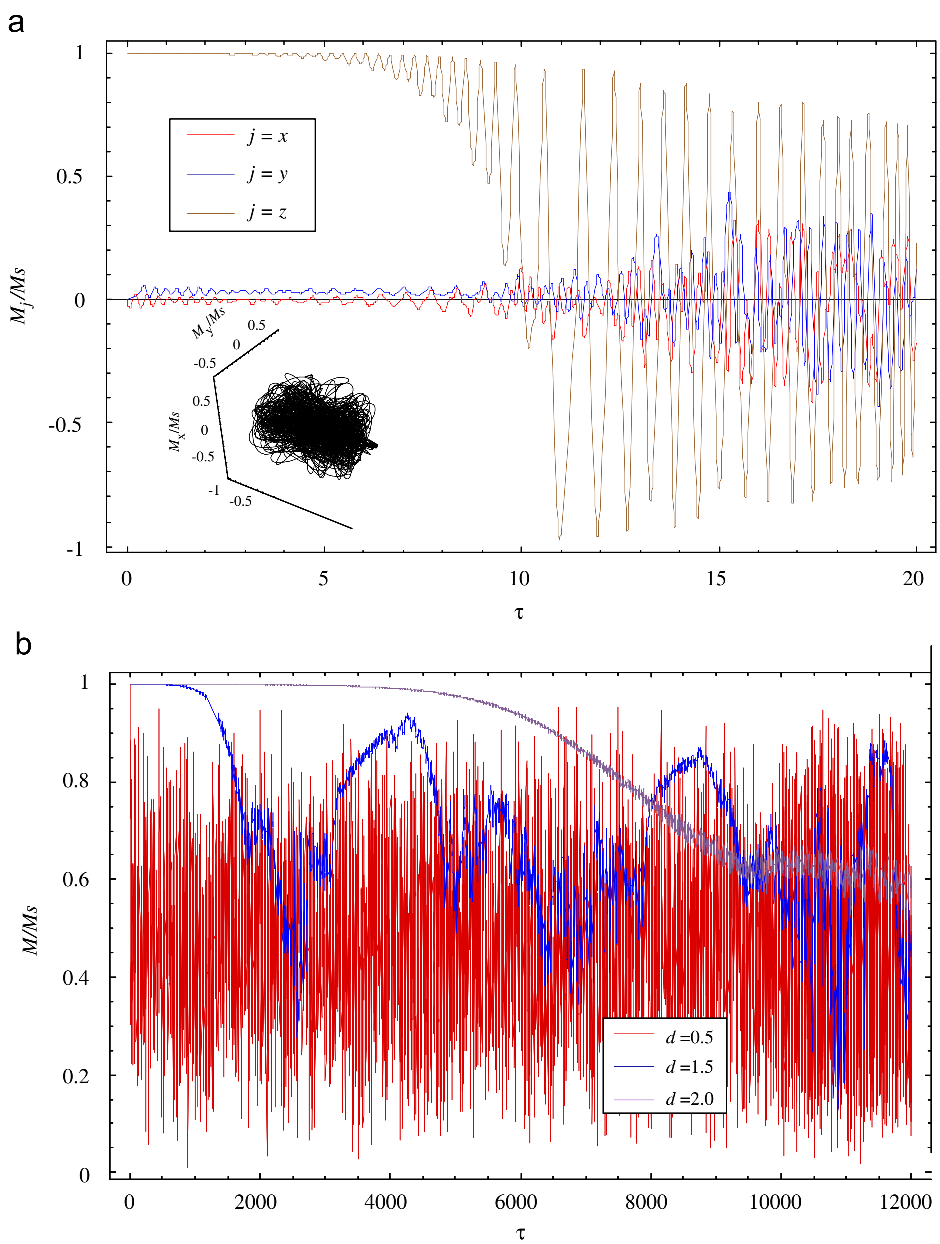

Fig. 2. (a) Cartesian component of the total magnetization, M_j/M_s, as a function of time for $h = 1m/(d^3H_0) = 15.625$ and $J/(mH_0) = 5$. The inset shows a 3D phase diagram of $\mathbf{M}$ for the same set of parameters. (b) Normalized magnitude of the total magnetization, M/M_s, as a function of time for different values of d at $h = 1$ and $J/(mH_0) = 1$.

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$\mathbf{M} \times \mathrm{d}\mathbf{M}/\mathrm{d}t$ [10], because it preserves the modulus of $\mathbf{M}$ as a constant. Moreover, from Eq. (5) it is clear that the magnitude of the total magnetization is not conserved, it is fluctuating in time.

3. Numerical results

In order to integrate numerically Eq. (4), we express them in a dimensionless form; for this purpose we introduce the following new variables $\mathbf{e}^j = \mathbf{m}^j/m, \mathbf{h}^i_{\mathrm{eff}} = \mathbf{H}^i_{\mathrm{eff}}/H_0$, and $\tau = t/(\gamma H_0)$, where H_0 is the magnitude of a reference magnetic field. The initial conditions for the magnetic moments are selected in one particular saturation condition $\mathbf{e}^1_0 = \mathbf{e}^2_0 = \mathbf{e}^3_0 = \mathbf{e}^4_0 = \hat{\mathbf{z}}$ and we impose that $\mathbf{h} = h(1/\sqrt{2}, 1/\sqrt{2}, 0)$. The geometrical ring condition is $\mathbf{r}_i = d(\cos(\pi(i+1)/2), \sin(\pi(i+1)/2), 0)$. We will assume that, the four magnetic particles are identical. An order four Runge–Kutta numerical method was used to solve our 12 differential equations in the Cartesian coordinates given in Eq. (4). For testing our numerical method we calculate each individual magnetic moment with more than 35 significant numbers, as it is shown in Fig. 1(a).

Fig. 1 shows the time evolution of the fourth magnetic moment, $\mathbf{e}^4$, for $h = 1$ and $m/(d^3 H_0) = 15.625$. Fig. 1(a) corresponds to the three Cartesian components and the modulus of $\mathbf{e}^4$, as function of time at $Jm/H_0 = 4$. We observe that all the components exhibit a non-periodic behavior; in particular, the x and y components increase their amplitudes and the z component is taken off of its equilibrium state and produces an irregular behavior. Fig. 1(b) shows the z component of $\mathbf{e}^4$ as a function of time at $h = 1$ and $m/(d^3 H_0) = 15.625$ for different values of J. We note that, when $J = 0$ z component always fluctuates, nevertheless when the exchange interaction is taken into account its fluctuation tends to diminish; moreover, when J increases e^4_z remains constant in a time interval and after that e^4_z exhibits an irregular behavior. In addition, for a fixed value of J and when the intensity of the dipolar interaction decreases, the time interval in which this component stays constant increases. We remark that, its dynamical behavior strongly depends on the relative interaction coupling constants.

Fig. 2 shows the time evolution of the total magnetic moment, $\mathbf{M}$, at $h = 1$. Fig. 2(a) shows the three Cartesian components of $\mathbf{M}$ as a function of time for $m/(d^3 H_0) = 15.625$ and $Jm/H_0 = 5$. Notice that, until $\tau \sim 10$ all the components fluctuate near their equilibrium states, afterward their amplitudes increases, a chaotic movement is revealed as it is shown in the inset. Fig. 2(b) shows the modulus of the total magnetization, M, as a function of time at $h = 1$ and $J/(mH_0) = 1$ for different values of d. Notice that, it presents a non-periodic behavior and is always a fluctuating function of time; so this figure confirm, numerically, the structure of Eq. (5). When the exchange interaction is the relevant term in Hamiltonian (3), M remains constant for some time interval, after that it exhibits a transition to a fluctuating function of time.

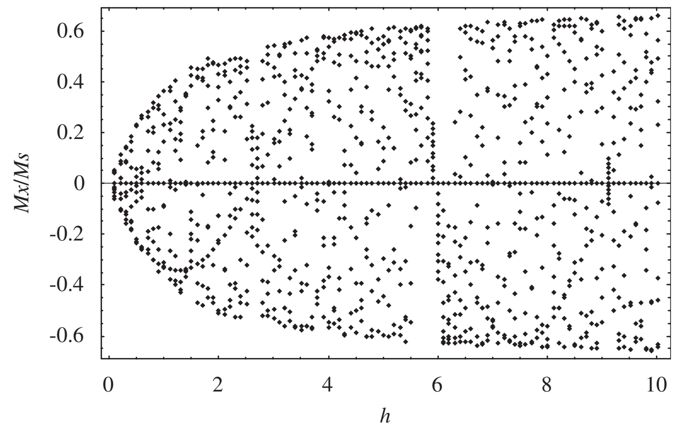

Fig. 3. Bifurcation diagram of the normalized x component of the total magnetization, M_x/M_s, as a function of h for $m/(d^3 H_0) = 1$ and $J/(mH_0) = 1$.

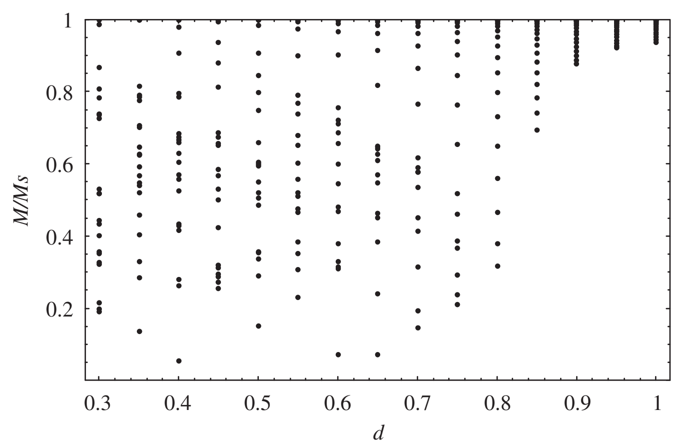

Fig. 4. Bifurcation diagram of the normalized total magnetization modulus, M/M_s,as a function of d for $h = 1$ and $J/(mH_0) = 1$.

Now let us analyze the parameter space, for this propose we calculate numerically some full bifurcation diagrams. Fig. 3 shows the bifurcation diagrams for the x component of $\mathbf{M}$ as a function of h for $m/(d^3 H_0) = 1$and $J/(mH_0) = 1$. This diagram is similar to the diagrams of doubling period; therefore, we can suggest that this system will exhibit a chaotic behavior when h increases. Fig. 4 shows the bifurcation diagram for the modulus of the total magnetization, M, as a function of d for $h = 1$ and $J/(mH_0) = 1$ We remark that, when d increases, M becomes stable only at unity, and it is a physical correct result, because when strength of dipolar interaction decreases the modulus of total magnetization must tend to a constant value.

4. Conclusions

The spin dynamics of four identical interacting magnetic particles considering dipole–dipole and exchange interactions in the presence of an applied magnetic field on ring configuration was analyzed. Due to the two kinds of interactions, two time scales arise: a short time scale related to the dipolar interaction and a long time scale associated with the exchange interaction. As a result of the dipolar interaction, the modulus of the total magnetization is a fluctuating function of time. If we only consider the dipole interaction, the modulus of the total magnetization presents a non-periodic structure; however, when the exchange interaction is taken into account, it remains constant for some time interval, after that it exhibits a transition to a fluctuating function of time. Finally, we remark that, when the strength of dipolar interaction decreases the normalized modulus of total magnetization tend to the unity.

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